# HOSSAM GHANEM

## مسائل النهايات

### باستخدام التعريف

$\lim_{x\to a} f(x) = L$

Let $|f(x) - L| < \varepsilon$

↓ عمليات جبرية

$|x - a| < \frac{\varepsilon}{k}$

Take $\delta = \frac{\varepsilon}{k}$

$\therefore \ \forall\, \varepsilon > 0 \quad \exists\, \delta > 0$

such that $|x - a| < \delta$

$\therefore \lim_{x\to a} f(x) = L$

العمليات الجبرية هي الجمع و أخذ العامل المشترك

| | | |
|---|---|---|
| $\forall$ | For all | لكل |
| $\varepsilon$ | Epsilon | ابسلون |
| $\delta$ | Delta | دلتا |
| $\exists$ | There exists | يوجد |

### تحتوي على $|g(x)|$ أو $\sqrt{x^2}$

$\lim_{x\to a} |g(x)|$

↓

$\lim_{x\to a^-} |g(x)| = L_1$

$\lim_{x\to a^+} |g(x)| = L_2$

- $L_1 = L_2$: $\lim_{x\to a} |g(x)| = L_1$
- $L_1 \neq L_2$: D·N·E

### النهاية تسعى إلي صفر الجذر التربيعي

$\lim_{x\to a^-} \sqrt{x-a}$ D · N · E

$\lim_{x\to a^+} \sqrt{x-a} = 0$

$\lim_{x\to a} \sqrt{x-a}$ D · N · E

### تعويض مباشر

- → $\frac{L}{\infty}$
- → $\frac{0}{0}$ ، $\frac{\infty}{\infty}$ ، $0 \cdot \infty$ → تحليل / مرافق / قسمة مطولة → تعويض → $\frac{L}{\infty}$
- → $\infty - \infty$ → توحيد مقامات → تحليل → تعويض

# (1) 2.4 *Definition of limit*

| Example 1<br>39 July 3, 2004 | Use the definition of the limit to show that $\lim_{x \to 2}(3x-5)=1$ |
|---|---|

## Solution

$\lim_{x \to 2}(3x-5)=1$

Let $\epsilon > 0$ such that $|f(x)-L| < \epsilon$

$|3x-5-1| < \epsilon$

$|3x-6| < \epsilon$

$3\,|x-2| < \epsilon$

$|x-2| < \frac{\epsilon}{3}$

Take $\delta = \frac{\epsilon}{3}$

$\therefore\ \forall\, \epsilon > 0 \quad \exists\, \delta > 0$ such that $|x-2| < \delta$

$\therefore\ \lim_{x \to 2}(3x-5)=1$

| Example 2<br>52 April 9, 2009 A | Use the definition of the limit to show that $\lim_{x \to -3}(2x-1) = -7$ |
|---|---|

## Solution

$\lim_{x \to -3}(2x-1) = -7$

Let $\epsilon > 0$ such that $|f(x)-L| < \epsilon$

$|2x-1-(-7)| < \epsilon$

$|2x+6| < \epsilon$

$|x+3| < \frac{\epsilon}{2}$

$|x-(-3)| < \frac{\epsilon}{2}$

Take $\delta = \frac{\epsilon}{2}$

$\therefore\ \forall\, \epsilon > 0 \quad \exists\, \delta > 0$ such that $|x-(-3)| < \delta$

$\therefore\ \lim_{x \to -3}(2x-1) = -7$

| Example 3<br>40 October 28, 2004 A | Use the definition of the limit to show that $\lim_{x \to 4}(7 - 2x) = -1.$ |
|---|---|

## Solution

$a - b = -(b - a)$
$|a - b| = |b - a|$

$\lim_{x \to 4}(7 - 2x) = -1$
let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$
$|7 - 2x - (-1)| < \epsilon$
$|8 - 2x| < \epsilon$
$|2x - 8| < \epsilon$
$2|x - 4| < \epsilon$
$|x - 4| < \frac{\epsilon}{2}$

Take $\delta = \frac{\epsilon}{2}$

$\therefore \ \forall \epsilon > 0 \quad \exists \delta > 0$ such that $|x - 4| < \delta$
$\therefore \lim_{x \to 4}(7 - 2x) = -1$

| Example 4<br>53 July 18, 2009 A | Use the definition of the limit to show that $\lim_{x \to -2}(1 - 4x) = 9$ |
|---|---|

## Solution

$-a - b = -(b + a)$
$|-a - b| = |a + b|$

$\lim_{x \to -2}(1 - 4x) = 9$
let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$
$|1 - 4x - 9| < \epsilon$
$|-4x - 8| < \epsilon$
$|4x + 8| < \epsilon$
$4|x + 2| < \epsilon$
$|x + 2| < \frac{\epsilon}{4}$
$|x - (-2)| < \frac{\epsilon}{4}$
take $\delta = \frac{\epsilon}{4}$
$\therefore \ \forall \epsilon > 0 \quad \exists \delta > 0$ such that $|x - (-2)| < \delta$
$\therefore \lim_{x \to -2}(1 - 4x) = 9$

| Example 5<br>46 July 5, 2007 | Use the $(\epsilon, \delta)$ definition of the limit to show that $\lim_{x\to 1}\left(3-\frac{1}{2}x\right)=\frac{5}{2}$ |
|---|---|

## Solution

$\lim_{x\to 1}\left(3-\frac{1}{2}x\right)=\frac{5}{2}$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$\left|3-\frac{1}{2}x-\frac{5}{2}\right| < \epsilon$

$|6 - x - 5| < 2\,\epsilon$ (بضرب الطرفين في 2)

$|1 - x| < 2\epsilon$

$|x - 1| < 2\epsilon$

take $\delta = 2\epsilon$

$\therefore\ \forall\, \epsilon > 0 \quad \exists\, \delta > 0$ such that $|x - 1| < \delta$

$\therefore \lim_{x\to 1}\left(3-\frac{1}{2}x\right)=\frac{5}{2}$

| Example 6<br>8 October 28, 1993 | Use the definition of the limit to show that $\lim_{x\to -\frac{1}{3}}(-2x+1)=\frac{5}{3}$ |
|---|---|

## Solution

$\lim_{x\to -\frac{1}{3}}(-2x+1)=\frac{5}{3}$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$\left|-2x+1-\frac{5}{3}\right| < \epsilon$

$|-6x + 3 - 5| < 3\,\epsilon$ (بضرب الطرفين في 3)

$|-6x - 2| < 3\epsilon$

$|6x + 2| < 3\epsilon$

$6\left|x+\frac{2}{6}\right| < 3\epsilon$

$\left|x+\frac{1}{3}\right| < \frac{3\epsilon}{6}$

$\left|x-\left(-\frac{1}{3}\right)\right| < \frac{\epsilon}{2}$

take $\delta = \frac{\epsilon}{2}$

$\therefore\ \forall\, \epsilon > 0 \quad \exists\, \delta > 0$ such that $\left|x-\left(-\frac{1}{3}\right)\right| < \delta$

$\therefore \lim_{x\to -\frac{1}{3}}(-2x+1)=\frac{5}{3}$

| | |
|---|---|
| **Example 7**<br>9 November 1993 | Use the definition of the limit to show that $\lim_{x \to -2} (3x - 7) = -13$ |

## Solution

$\lim_{x \to -2} (3x - 7) = -13$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$|3x - 7 + 13| < \epsilon$

$|3x + 6| < \epsilon$

$3|x + 2| < \epsilon$

$|x + 2| < \frac{\epsilon}{3}$

$|x - (-2)| < \frac{\epsilon}{3}$

take $\delta = \frac{\epsilon}{3}$

$\therefore \forall \epsilon > 0 \quad \exists \delta > 0$ such that $|x - (-2)| < \delta$

$\therefore \lim_{x \to -2} (3x - 7) = -13$

| | |
|---|---|
| **Example 8**<br>11 March 31, 1994 | Use the definition of the limit to show that $\lim_{x \to -4} \left(-\frac{x}{2} + 3\right) = 5$ |

## Solution

$\lim_{x \to -4} \left(-\frac{x}{2} + 3\right) = 5$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$\left|-\frac{x}{2} + 3 - 5\right| < \epsilon$

$|-x + 6 - 10| < 2\epsilon$ ( بالضرب في 2 )

$|-x - 4| < 2\epsilon$

$|x + 4| < 2\epsilon$

$|x - (-4)| < 2\epsilon$

take $\delta = 2\epsilon$

$\therefore \forall \epsilon > 0 \quad \exists \delta > 0$ such that $|x - (-4)| < \delta$

$\therefore \lim_{x \to -4} \left(-\frac{x}{2} + 3\right) = 5$

| Example 9<br>5 April 8, 1993 | Use the definition of the limit to show that $\lim_{x \to \frac{1}{2}} (-2x + 5) = 4$ |
|---|---|

## Solution

$\lim_{x \to \frac{1}{2}} (-2x + 5) = 4$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$|-2x + 5 - 4| < \epsilon$

$|-2x + 1| < \epsilon$

$|2x - 1| < \epsilon$

$2\left|x - \frac{1}{2}\right| < \epsilon$

$\left|x - \frac{1}{2}\right| < \frac{\epsilon}{2}$

take $\delta = \frac{\epsilon}{2}$

$\therefore\ \forall \epsilon > 0 \quad \exists \delta > 0$ such that $\left|x - \frac{1}{2}\right| < \delta$

$\therefore \lim_{x \to \frac{1}{2}} (-2x + 5) = 4$

| Example 10<br>31 October 31, 2000 | Use the definition of the limit to show that $\lim_{x \to \frac{2}{3}} (4 - 5x) = \frac{2}{3}$ |
|---|---|

## Solution

$\lim_{x \to \frac{2}{3}} (4 - 5x) = \frac{2}{3}$

let $\epsilon > 0$ such that $|f(x) - L| < \epsilon$

$\left|4 - 5x - \frac{2}{3}\right| < \epsilon$

$|12 - 15x - 2| < 3\epsilon$

$|-15x + 10| < 3\epsilon$

$|15x - 10| < 3\epsilon$

$\left|x - \frac{10}{15}\right| < \frac{3\epsilon}{15}$

$\left|x - \frac{2}{3}\right| < \frac{\epsilon}{5}$

take $\delta = \frac{\epsilon}{5}$

$\therefore\ \forall \epsilon > 0 \quad \exists \delta > 0$ such that $\left|x - \frac{2}{3}\right| < \delta$

$\therefore \lim_{x \to \frac{2}{3}} (4 - 5x) = \frac{2}{3}$

# Homework

| | | | |
|---|---|---|---|
| 1 | Use the definition of limit to prove that | $\lim_{x\to -2} 7x+2=-12$ | 10 October 27, 1994 |
| 2 | Use the definition of the limit to show that | $\lim_{x\to \frac{1}{3}}(3x-4)=-3$ | 15 July 15, 1996<br>19 November 1, 1997 |
| 3 | Use the definition of limit to prove that | $\lim_{x\to 2}(3x-1)=5$ | 16 November 2, 1996 |
| 4 | Use the definition of the limit to show that | $\lim_{x\to -\frac{1}{3}}(6x+1)=-1$ | 17 March 27, 1997 |
| 5 | Use the definition of limit to prove that | $\lim_{x\to 1}(-3x+5)=2$ | 21 March 3, 1998 |
| 6 | Use the definition of the limit to show that | $\lim_{x\to \frac{1}{2}}(-6x+1)=-2$ | 22 July 18, 1998 A |
| 7 | Use the definition of limit to prove that | $\lim_{x\to -1}(3x+9)=6$ | 29 Feb 24, 2000 |
| 8 | Use the definition of the limit to show that | $\lim_{x\to -1}(2x+5)=3$ | 2 November 9, 1989 |
| 9 | Use the $\epsilon,\delta$ definition of limit to show that | $\lim_{x\to 2}(5-2x)=1.$ | 43 June 28, 2008 |
| 10 | Use the definition of the limit to show that | $\lim_{x\to \frac{1}{2}}(4x-5)=-3$ | 32 August 02, 2008 |
| 11 | Use the definition of the limit to show that | $\lim_{x\to -4}\left(-\frac{x}{2}+3\right)=5$ | 11 March 31, 1994 |
| 12 | Use the definition to prove that | $\lim_{x\to 6}\left(9-\frac{1}{6}x\right)=8$ | 37 June 6, 2010 |